RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2613**

Statistics 1

Thursday **9 JUNE 2005** Morning 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 A bookshop has a selection of 60 different mathematics books. The numbers of chapters contained in these books are summarised in the table.

Number of chapters	3–5	6–8	9–11	12–16	17–21	22–30
Number of books	4	6	12	14	14	10

(i) Calculate estimates of the mean and standard deviation of the number of chapters per book. [5]

(ii) Why is it not possible to obtain exact values for the mean and standard deviation from the data in the table? [1]

In fact, the exact values of the mean and standard deviation of the number of chapters per book are 14.7 and 6.1. Use these exact values for the remainder of the question.

(iii) For each of the two statements below, state with reasons whether it is **definitely true**, **definitely false** or **possibly true**.

Statement 1 There is at least one outlier below the mean. [3]

Statement 2 There is at least one outlier above the mean. [3]

The number of pages, p, in a book is modelled by

$$p = 20x + 15,$$

where x is the number of chapters in the book.

(iv) Calculate the mean and standard deviation of the number of pages, as given by this model. [3]

2 Midchester Rovers and Southpool United are two teams in a football league. Previous experience shows that, when these two teams play each other, the numbers of goals scored are independent and have the following probabilities.

Number of goals	0	1	2	3
Midchester Rovers	0.2	0.3	0.35	0.15
Southpool United	0.4	0.5	0.1	0

The two teams play each other. Find the probabilities of the events in parts **(i)** to **(v)**.

(i) Midchester win the game 1 – 0. [1]

(ii) One of the teams wins the game 2 – 0. [2]

(iii) The game ends in a draw. [3]

(iv) Southpool win the game. [3]

(v) Midchester score no goals given that Southpool win the game. [3]

The probability that Southpool fail to score in k successive games against Midchester is greater than 1%.

(vi) Find the greatest possible value of k. [3]

3 A production line operates for 5 days each week and produces 200 components per day. As part of a quality control system, 100 components have to be examined per week.

(i) A systematic sample of size 20 is taken each day. Describe how this could be done. [2]

(ii) An alternative method would be to take a systematic sample of size 100 on just one day each week. Give one advantage and one disadvantage of this method compared with the method described in part **(i)**. [2]

(iii) A further method would be to take a simple random sample of 20 components each day. Describe how this could be done using a random number generator which produces whole number values from 000 to 999. [3]

A quality control manager checks the work of his team by testing some of the components that they have examined. He decides to test 5 components, chosen at random, from 15 examined by Ken, a member of the team.

(iv) How many different selections of 5 components can the manager make? [2]

(v) Ken has made mistakes examining 2 of these 15 components. Find the probability that the manager will

(*A*) test neither of these 2 components, [3]

(*B*) test exactly 1 of these 2 components. [3]

4 Over a number of years, the proportion of pupils at Westport High School achieving grade C or better in their mathematics GCSE has been 55%.

A random sample of 20 GCSE pupils at Westport High School is taken. Assuming that the proportion of pupils achieving grade C or better in mathematics remains 55%, find, for this sample,

(i) the probability that exactly 13 achieve grade C or better, [3]

(ii) the probability that 8 or more achieve grade C or better, [2]

(iii) the expected number of pupils achieving grade C or better. [2]

In the past year, the mathematics department at Westport High School made a trial of a new homework policy for GCSE pupils, intended to increase the proportion who achieve grade C or better. This policy was used for 20 pupils, who may be regarded as a random sample.

16 of the 20 pupils achieve grade C or better.

(iv) Write down the hypotheses you would use to test whether the new homework policy has been successful. Explain why your alternative hypothesis takes the form it does. [3]

(v) Carry out the test at the 5% level, showing all your working. [5]

Mark Scheme 2613
June 2005

Question 1

(i)	Mid f mf m^2f 4 4 16 64 7 6 42 294 10 12 120 1200 14 14 196 2744 19 14 266 5054 26 10 260 6760 60 900 16116 mean = 900/60 = 15 chapters variance = $\frac{1}{60}\left(16116-\frac{900^2}{60}\right)$ = 43.6 $\rightarrow$ standard deviation = 6.60(3) with n-1 divisor = 6.65(9)	B1 mid points B1 correct m^2f A1 mean M1 sd method A1 correct sd	5
(ii)	Exact values not given	E1	1
(iii)	1: Definitely false 14.7 – 2(6.1) = 2.5 There are no values below 2.5 2: Possibly true 14.7 + 2(6.1) = 26.9 There may be values above 26.9	A1 answer M1 for 2.5 E1 for comment A1 answer M1 for 26.9 E1 for comment	6
(iv)	Mean = 20(14.7) + 15 = 309 pages Sd = 20(6.1) = 122 pages	B1 M1 A1	3
			15

Question 2

(i)	P(M win 1 – 0) = (0.3)(0.4) = 0.12	 B1	1
(ii)	P(Game ends 2 – 0) = (0.35)(0.4) + (0.2)(0.1) = 0.16	 M1 sum of 2 pairs A1	2
(iii)	P(neither team wins) = (0.2)(0.4)+(0.3)(0.5)+(0.35)(0.1) or (0.2)(0.6) + (0.3)(0.1) = 0.265	 M1 for 1 pair M1 for all pairs A1	3
(iv)	P(S scores more goals) = (0.1)(0.2)+(0.1)(0.3)+(0.5)(0.2) = 0.15	 M1 for 1 pair M1 for 3 pairs A1	3
(v)	P(M scores 0 given S wins) $= \frac{(0.1)(0.2)+(0.5)(0.2)}{0.15}$ = 0.8	 M1 numerator M1 denominator A1	3
(vi)	$0.4^k > 0.01$ $0.4^5 = 0.01024$ and $0.4^6 = 0.004096$ → maximum value of k is 5	M1 for inequality M1 for 0.01024 and 0.004096 A1	3
			15

Question 3

(i)	Randomly select start value 1 - 10 Then select every 10^{th} component until 20 have been selected.	E1 E1	2
(ii)	Advantage – cheaper/simpler to sample on just one day. Disadvantage – any problem could be missed for several days. Or any other sensible suggestion.	E1 E1	2
(iii)	If number generated is 001-200, select that component. If number generated is 201-000, subtract any whole 200's, or any correct strategy for numbers outside 001 – 200. Discard any repeated numbers.	E1 E1 E1	3
(iv)	$\binom{15}{5} = 3003$	M1 A1	2

(v)	(*A*) Prob = $\dfrac{\binom{13}{5}}{\binom{15}{5}}$ or $\dfrac{13.12.11.10.9}{15.14.13.12.11}$ = 1287/3003 = 3/7 (0.429) OR (A) $\binom{5}{0}\times\binom{10}{2}/\binom{15}{2} = \dfrac{3}{7}$ OR (A) $\dfrac{10}{15}\times\dfrac{9}{14}=\dfrac{3}{7}$	M1 numerator M1 denominator A1 M1 numerator M1 denominator A1 cao M1 for 1st fraction M1 for 2nd fraction A1 cao	

	(B) Prob = $\dfrac{\binom{13}{4}\binom{2}{1}}{\binom{15}{5}}$ or $\dfrac{5.13.12.11.10.2}{15.14.13.12.11}$ $= \dfrac{(2)(715)}{3003}$ $= 0.476 \quad = 10/21$ OR (B) $\binom{5}{1}\times\binom{10}{1}/\binom{15}{2} = \dfrac{10}{21}$ OR (B) $\dfrac{5}{15}\times\dfrac{10}{14}\times 2 = \dfrac{10}{21}$	M1 numerator M1 denominator A1 M1 numerator M1 denominator A1 cao M1 for 1st fraction M1 for 2nd fraction A1 cao	6
			15

Question 4

(i)	P(X = 13) = P(X ≤ 13) – P(X ≤ 12) = 0.8701 – 0.7480 = 0.1221	M1 M1 A1	3
(ii)	P(X ≥ 8) = 1 – P(X ≤ 7) = 1 – 0.0580 = 0.942	M1 A1	2
(iii)	Expected number pupils = 20(0.55) = 11	M1 A1	2
(iv)	Let p be the probability of a pupil achieving a grade C or better H_0: p = 0.55 H_1: p > 0.55 Because dept. looking for improvement	 B1 B1 E1	3
(v)	X ~ B(20, 0.55) P(X ≥ 16) = 1 – P(X ≤ 15) = 1 – 0.9811 = 0.0189 This is less than 5% so reject H_0 Conclude proportion with C or better has increased.	 M1 for correct tail M1 for method A1 M1 comparison with 5% E1 comment in context	5
			15

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General Comments

The examination attracted a broad range of candidates from across the ability spectrum. There were many who were comfortable with the content of the paper and were able to score highly but equally there was a cohort for whom the paper was beyond reach.

Generally, the responses to question 1 were sound but there were many protracted solutions seen in the attempt of the last part of the question. Question 2 was, without doubt, the most successfully answered question with many gaining full or close to full marks.

The work on sampling methods in question 3, with the related probability calculations, caused a noticeable dip in performance by almost all of the candidates. Whilst a sizeable proportion of the candidates made good headway in question 4 there were many erroneous methods and misconceptions prevalent. It was disappointing to see that candidates had trouble with the hypothesis test on the Binomial distribution. Only a few years ago the examiners felt that candidates were beginning to improve in this part of the specification. There are still too many candidates using point rather than tail probabilities to construct their argument.

The presentation of the solutions was generally pleasing with only a small handful of scripts resembling battlefields. Probabilities expressed as percentages have all but now disappeared. As a matter of protocol, candidates should be made aware that they risk losing marks by showing no working. It was not uncommon to see the incorrect answers, with no working, to a question e.g. mean =14.87, standard deviation = 6.7 followed by the words …. (Calc used). The examiners cannot be expected to unpick such a response to find hidden method marks.

Comments on Individual Questions

1) **Estimates of the mean and standard deviation of a discrete grouped data set. Reliability of the calculations. Outlier testing. Linear coding of the mean and standard deviation.**

(i) Almost all candidates were able to find and use the mid-points of the classes to estimate the mean. There was a small minority who thought they had to use the class widths instead of the mid points. Many were able to continue successfully to find the standard deviation but there were a worrying number who calculated f^2x, or even worse, $(fx)^2$ and tried to use these in the standard deviation formula. Candidates who used $f(x-\bar{x})^2$ often made careless errors along the way.

(ii) A little more than the response 'because the data are grouped' was required to earn the mark here for explaining why the mean and standard deviation were not exact values. Some indication that the original raw data had been absorbed into the table or that mid points were being used to represent a class width was needed to clinch the mark.

(iii) There were some very sensible attempts to this part of the question. Most were able to calculate the mean $\pm$ 2 standard deviations and identify the outliers but the examiners did see 1.5 and even 3 in place of 2. The statements and consequent reasoning were usually correct but some insisted that there **had to be** at least 1 outlier above 26.9 rather than there **may be** values above 26.9. Some candidates thought erroneously that they could round the lower outlier of 2.5 to 3 and tried to argue that 'this value was now inside the data'. Candidates did lose marks for (a) not using 14.7 and 6.1 as requested in the question, preferring instead to use the original mean of 15 and standard deviation of 6.6 or (b) attempting to answer the question qualitatively without recourse to any numerical evidence or calculation. The latter group suffered the most penalties.

(iv) Those who realised that they had to substitute the mean of 14.7 into $p = 20\bar{x} + 15$ and the standard deviation of 6.1 into $sd_y = 20\ sd_x$ had the solution out in two lines. However, many proceeded along protracted lines and tried to convert the original data (given as chapters) into pages by using the formula, thus wasting an inordinate amount of time. Often the calculations faltered due mainly to not realising that the original frequencies were required. It was not uncommon to see nearly a page of work with, alas, the incorrect final answers.

2) **Probability question on football scores including conditional probability and the solving of an inequality in relation to the game.**

(i) Very well answered with only a small number giving 0.3 instead of 0.12 as the answer.

(ii) Well attempted. Almost all achieved 0.16 as the answer with a small number multiplying the answer by 2, believing that the order was germane to the question.

(iii) Once again, a very positive response with 0.265 seen regularly.

(iv) Invariably correct but occasionally one of the terms was curiously missing.

(v) Most recognised the need for a conditional probability calculation but many solutions stopped short of this with only the 0.12 being calculated (required for the numerator). A generous follow through from part (iv) was allowed for the denominator.

(vi) Very few candidates were able to set up the initial inequality of $0.4^k > 0.01$ with alternatives of $0.4^k = 0.01$ or $0.4k = 0.01$ or even $0.2^k > 0.01$ being regularly seen. For those using trial and improvement it was essential that they tested 0.4^5 **and** 0.4^6 in order to gain the method mark. Some only went as far as 0.4^5 and then declared unequivocally that k = 5 **must** be the answer. Such faltering logic was penalised. There was a fair minority who thought that the question was asking for $p(X \geq k) > 1\%$.

3) Systematic Sampling of components. Comparison of sampling procedures. Using random numbers to select a sample. Calculation of the number of selections with associated probability methods.

(i) Most candidates scored at least one of the two marks available here. Whilst most realised that a selection of every 10th component was necessary, fewer appreciated that a random starting value between 1 and 10 was needed for the selection of the first component. For those deciding to choose a starting point (above 10) there had to be a clear indication that the cycle was being completed if every 10th component was mentioned. A small minority of candidates thought that the systematic sample was to do with the times of the day being split up before the components were selected.

(ii) Many candidates gave sensible answers to this part of the question, realising that for the advantage a response along the lines of cheaper/simpler or less time consuming was required. For the disadvantage, many realised that such a form of sampling on one day only was not necessarily representative of the rest of the week.

(iii) There was a variety of responses to this part. At a simplistic level some candidates thought they could select the 200 components by using the random numbers 000 to 999 without any further ado or consideration of random numbers greater than 200. This gained no credit. At the next level, a sizeable majority stated that if the random number generated was 001 to 200 (and discarding numbers greater than 200) then the components that had been allocated these numbers could be selected. This deserved 1 out of the 3 marks available. The more discerning candidates realised that they had to do something with the random numbers greater than 200. Various acceptable methods were **either** allocating blocks of numbers to each component e.g. 000 – 004 corresponded to component 1; 005 - 009 corresponded to component 2 …… 995 – 999 corresponded to component 200 **or** dividing each generated random number by 5 and rounding up/down to create a number back in the range 1 to 200 **or** even slicing layers of 200 from the generated random number e.g. if the random number generated was 201 – 400 then subtract 200; if the random number generated was 401 – 600 then subtract 400 etc. The final mark, that very few earned, was for realising that repeated numbers must be discarded. One wonders about the definition of a random sample that one candidate gave: 'A random sample is the random a sample can get but it will never always be 100% random'.

(iv) Invariably answered correctly with $^{15}C_5 = 3003$ being seen.

(v) This part of the question proved to be difficult for many candidates with many believing that a binomial probability calculation was required which, of course, it was not. The correct response to (A) of either $\frac{\binom{13}{5}}{\binom{15}{5}}$ or $\frac{13.12.11.10.9}{15.14.13.12.11}$ or more succinctly $\frac{10}{15}\cdot\frac{9}{14} = \frac{3}{7}$ was seldom seen. Instead, the examiners saw on many occasions $\frac{13}{15}\cdot\frac{12}{14}$ or $\binom{15}{0}\left(\frac{2}{15}\right)^0\left(\frac{13}{15}\right)^{15}$ or marginally better $\binom{5}{0}\left(\frac{2}{15}\right)^0\left(\frac{13}{15}\right)^5$. Only the latter solution gained some credit under 'special case'. Similarly, in part (B) the expected working of $\frac{\binom{13}{4}\binom{2}{1}}{\binom{15}{5}}$ or $\frac{5.13.12.11.10.2}{15.14.13.12.11}$ or more succinctly $\frac{5}{15}\cdot\frac{10}{14}\times 2$ was often replaced by the incorrect versions of $\frac{2}{15}\cdot\frac{13}{14}\times 2$ or $\binom{15}{1}\left(\frac{2}{15}\right)^1\left(\frac{13}{15}\right)^{14}$ or marginally better $\binom{5}{1}\left(\frac{2}{15}\right)^1\left(\frac{13}{15}\right)^4$. Again, only the last showing gained some credit under 'special case'. A fair proportion of candidates omitted this part of the question.

4) **Use of the Cumulative Binomial tables or formula in the context of examination passes. Expectation of the Binomial distribution. One tailed hypothesis test on the Binomial distribution.**

(i) A surprising number of candidates fell at the first hurdle. Many were unable to use the binomial tables correctly to find $p(X = 13)$. Some mistakenly believed that $p(X = 13)$ was found from $p(X \leq 14) - p(X \leq 12)$.

(ii) Again, many errors were seen in the calculation of $p(X \geq 8)$ with many believing it was found from 1- $p(X \leq 8)$ or even $1 - p(X = 7)$. On occasions the examiners wondered whether some candidates had access to the binomial tables particularly when candidates resorted to protracted methods by calculating $\sum_{9}^{20} p(X = x)$ or even $1 - \sum_{0}^{7} p(X = x)$.

(iii) Invariably correct but curiously many went on to calculate $p(X = 11)$ which was **not** asked for in the question.

(iv) The statements for H_0 and H_1 were usually given in the correct form but there are still candidates who squander valuable marks by using a sloppy notation. As it has been mentioned in almost every previous report it is **NOT** acceptable to write $H_0 = 0.55$ or even $H_0 : p(x = 0.55)$. Such notations are penalised. The explanation of 'why the alternative hypothesis took the form it did' was usually well answered by most but this year's howler must go to the candidate who wrote 'because the **police** meant to increase the average number of pupils passing at grade C or above'. Certainly one alternative to present educational methods! The subsequent work on the hypothesis test was quite depressing with an inordinate amount of candidates favouring an argument involving point probabilities rather than a tail probability. Even those who knew they had to find $p(X \geq 16)$ often faltered by giving $1 - p(X \leq 16)$ instead of $1 - p(X \leq 15)$.